A New Method to Determine the Optimum Load of a Real Solar Cell using Special Trans Function Theory (STFT)

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Abstract- An analytical expression for optimum load for a real solar cell, having resistive and current leakage losses, has been obtained using special trans function theory (STFT) for solving the transcendental current-voltage relationship. Theoretically calculated value using STFT is compared with the experimental data as well as the values obtained by other methods. Impact of series resistance and shunt resistance on optimum load is also studied. The numerical results indicate that STFT method has the best accuracy over other available methods in literature.

Keywords- STFT, Optimum load, Solar Cell.

1. Introduction

Maximum power from a solar cell can be extracted only when it is operated with an optimum load. Even a small deviation of 1% from the optimum power transfer condition can cause a loss of output power by ~10% in solar cell panels operating under normal conditions [1,2]. The effect is even more pronounced at high intensities of illumination [8].

According to maximum power transfer theorem, for a linear circuit network, the maximum power transfer occurs at an optimal load having impedance equal to the complex conjugate of the source impedance [3]. However, the current-voltage characteristic of a solar cell is non-linear and transcendental in nature. To address the problem of determining the optimum load \( R_{Lmp} \) of a solar cell, researchers adopted different techniques [4-10]. Loeferski [4] and Wysocki and Rappaport [5] calculated the load impedance at the maximum power transfer point for an ideal solar cell where the series resistance is zero and the shunt resistance is infinite. Another method of estimating the value of \( R_{Lmp} \) from a set of typical current-voltage characteristics was presented by Rao and Padmanabhan [6] for a silicon solar cell used in terrestrial applications. A trial and error method was adopted by Govil [7] to obtain the maximum output condition for a p-n junction solar cell with a series resistance or a shunt resistance; also derived the expressions for the optimum load \( R_{Lmp} \). Kothari et al [8] obtained the expression for optimum load of a real solar cell, having both resistive and current leakage losses (taking into account both \( R_s \) and \( R_{sh} \)) using Lagrange’s method of undetermined multipliers. The study of Wyatt and Chua [9] reports a multiport maximum power theorem for nonlinear resistive networks presenting the optimal nonlinear optimal characteristics as a closed form expression involving the constitutive relation of the source. It is applied to the optimal load-matching problem for a solar cell. The method of Lambert W-function was used by Ding and Radhakrishnan [10] to get an exact explicit expression for optimum load of an illuminated solar cell containing the parasitic resistances.

Of late, Special Trans Function Theory (STFT) [11] has been used to determine the parameters of real solar cells [12,
In this communication, an analytical expression for optimum load of a real solar cell is obtained, based on the solution of the basic current-voltage characteristics of the cell using STFT by writing it in a form that complies with the properties of STFT. The possibility of getting different gradients in STFT makes it possible to analyze analytically any problem rigorously and is applicable for arbitrary non-linear forms [11-13], and so this analysis should hold for various types of solar cells. Comparisons are also made with the calculated results of other analytical methods available in literature.

2. Theory

The current-voltage relation of a real solar cell based on the single diode model under illumination can be written as

\[ I = I_0 \left( \exp \left( \frac{V - I R_s}{nV_{th}} \right) - 1 \right) + \frac{V - I R_s}{R_sh} - I_{ph} \]

where \( I \) and \( V \) are the current and voltage output of the cell respectively, \( V_{th} (=kT/q) \) is the thermal voltage, \( I_{ph} \) is the light generated photocurrent, \( n \) is the diode ideality factor, \( R_s \) and \( R_{sh} \) are the parasitic series and shunt resistances respectively. The explicit analytical expression for current in terms of \( V \) and the newly defined trans function \( (trans_s(D)) \) and the model’s parameters using STFT is [12]

\[ I = \left( \frac{V}{R_s} - \frac{I_{ph} + I_0}{1 + \frac{R_{sh}}{R_s}} \right) \left( nV_{th} \left( 1 + \frac{R_{sh}}{R_s} \right) \frac{trans_s(D)}{trans_s(D) - 1} \right) \]

or

\[ I = \left( \frac{I_{ph} + I_0}{1 + \frac{R_{sh}}{R_s}} - \frac{V}{R_s} \right) + \frac{nV_{th}}{R_s} \left( D \sum_{m=0}^{\infty} \frac{D^m (x - m)^m}{m!} \right) \]

The trans function is defined by \( (trans_s(D)) \)

\[ Z = (trans_s(D)) \]

where \( trans_s(D) \) is a new special trans function [11-13] defined as

\[ trans_s(D) = \lim_{x\to\infty} \left[ D \left( \frac{\phi_s(D,x)}{\phi_s(D,x+1)} \right) \right] \]

and

\[ \phi_s(D,x) = \sum_{m=0}^{\infty} \frac{D^m (x - m)^m}{m!} \]

so that

\[ \frac{\partial P}{\partial V}_{V=V_{mp}} = 0 \]

and

\[ \frac{\partial L}{\partial V}_{V=V_{mp}} = \frac{1}{R_{mp}} \]

Equation (12) is the general condition for maximum power from which \( V_{mp} \) can be calculated for given values of other parameters.

The inverse of the slope on the \( I-V \) characteristic at

\[ V = V_{mp} \text{ i.e., } \left( \frac{\partial L}{\partial V} \right)_{V=V_{mp}} = \frac{1}{R_{mp}} \]
To investigate the STFT method of determining the RLmp, it is applied to three different experimental solar cells (namely grey, blue and plastic solar cells) I-V characteristics based on the single diode model of a real solar cell (Table 1). 

The theoretical values of RLmp were evaluated using equation (13) with respect to the condition of equation (12). Using the data of Charles et al [14] and Phang et al [15], optimum loads of two silicon solar cells (grey solar cell and blue solar cell) were calculated. Similarly, the optimum load of a plastic solar cell was determined using its recently published experimental data reported in [16]. Calculated results are compared with experimental data and the values determined by other methods available in literature. Relative accuracy is also calculated to show the significance of the STFT method (Table 2).

The calculated results show that the STFT method has the best accuracy over other available methods in literature. In this scheme, substituting the values of the model parameters in equations (12) and (13), yield the value of RLmp.

It is worth to analyze the dependence of RLmp on the series resistance and the shunt resistance of the solar cell. Fig. 1 represents the effect of the series resistance Rs on the RLmp of various cells. The graphs are in consistent with the conclusions of Kothari et al [8] and Ding and Radhakrishnan [10], according to which, higher the series resistance, the greater its effect on the optimum load. Fig. 2 shows the effect of shunt resistance Rsh on RLmp. From the graph, it is observed that smaller the shunt resistance, the greater its effect on optimum load, in consistent with the conclusions of Kothari et al [8] and Ding and Radhakrishnan [10].

### 3. Results and Discussion

#### Table 1. Parameters of experimental solar cells used for calculating optimum load resistance

<table>
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<tbody>
<tr>
<td>Rs</td>
<td>77.69 (mΩ)</td>
<td>68.26 (mΩ)</td>
<td>8.59 (Ωcm²)</td>
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<td></td>
</tr>
<tr>
<td>Rsh</td>
<td>25.9 (Ω)</td>
<td>1000.0 (Ω)</td>
<td>197.24 (Ωcm²)</td>
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<td></td>
</tr>
<tr>
<td>Is</td>
<td>5.514 (μA)</td>
<td>0.1036 (μA)</td>
<td>13.6 (nAcm⁻²)</td>
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<td></td>
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<tr>
<td>Is</td>
<td>0.5610 (A)</td>
<td>0.1023 (A)</td>
<td>7.94 (mAcm⁻²)</td>
<td></td>
<td></td>
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<tr>
<td>Vsh</td>
<td>26.479 (mV)</td>
<td>25.875 (mV)</td>
<td>25.875 (mV)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>1.7168</td>
<td>1.5019</td>
<td>2.31</td>
<td></td>
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<tr>
<td>Vmp</td>
<td>0.390 (V)</td>
<td>0.437 (V)</td>
<td>0.548 (V)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Im</td>
<td>0.481 (A)</td>
<td>0.0925 (A)</td>
<td>4.71 (mAcm⁻²)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>307 (K)</td>
<td>300 (K)</td>
<td>300 (K)</td>
<td></td>
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</table>

#### Table 2. Comparison of RLmp values obtained by different methods

<table>
<thead>
<tr>
<th>Methods</th>
<th>Grey Solar Cell</th>
<th>Blue Solar Cell</th>
<th>Plastic Solar Cell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present method (STFT method)</td>
<td>RLmp</td>
<td>0.8108 (Ω)</td>
<td>4.724 (Ω)</td>
</tr>
<tr>
<td>Accuracy (%)</td>
<td>1.406</td>
<td>1.769</td>
<td>0.103</td>
</tr>
<tr>
<td>Lambert W-function method</td>
<td>RLmp</td>
<td>0.7985 (Ω)</td>
<td>4.638 (Ω)</td>
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<tr>
<td>Accuracy (%)</td>
<td>1.517</td>
<td>1.821</td>
<td>0.146</td>
</tr>
<tr>
<td>Lofer’s method</td>
<td>RLmp</td>
<td>1.5498 (Ω)</td>
<td>4.9036 (Ω)</td>
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<tr>
<td>Accuracy (%)</td>
<td>91.149</td>
<td>3.801</td>
<td>293.958</td>
</tr>
<tr>
<td>Govil’s method</td>
<td>RLmp</td>
<td>0.6459 (Ω)</td>
<td>4.0337 (Ω)</td>
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<tr>
<td>Accuracy (%)</td>
<td>20.334</td>
<td>14.612</td>
<td>76.712</td>
</tr>
<tr>
<td>Only Rs</td>
<td>RLmp</td>
<td>0.7951 (Ω)</td>
<td>4.6698 (Ω)</td>
</tr>
<tr>
<td>Accuracy (%)</td>
<td>1.940</td>
<td>1.147</td>
<td>4.411</td>
</tr>
<tr>
<td>Only Rsh</td>
<td>RLmp</td>
<td>0.7414 (Ω)</td>
<td>4.2192 (Ω)</td>
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<tr>
<td>Accuracy (%)</td>
<td>8.554</td>
<td>10.686</td>
<td>0.821</td>
</tr>
</tbody>
</table>

#### 4. Conclusion

A new analytical method to determine the optimum load of a real solar cell from its model parameters is described using the properties of STFT. The method involves no approximation. The effects of the series resistance and shunt resistance on the optimum load are studied which are in
consistent with the results reported in literature. The advantage of the STFT method is that the calculation time is independent of the response time, and the transcendental calculation scheme for current independent of the time domain. We may conclude that the method presented here is a powerful theoretical tool for the study of real solar cells.

Fig. 1. Effect of the series resistance \( R_s \) on the optimum load \( R_{\text{Lmp}} \) of a (a) grey solar cell, (b) blue solar cell and (c) plastic solar cell

Fig. 2. Effect of the shunt resistance \( R_{sh} \) on the optimum load \( R_{\text{Lmp}} \) of a (a) grey solar cell, (b) blue solar cell and (c) plastic solar cell
References


