Optimal Sizing of Energy Storage System in a Micro Grid Using the Mixed Integer Linear Programming

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Abstract- Battery, as an energy storage system, plays an important role in operation of micro-grids (MG). This paper presents a new analytical cost-based approach to optimal sizing of battery energy storage systems (BESS) to reduce the operational and total costs of MGs. To do so, a unit commitment problem must be solved to obtain the optimal schedule of units, and then the proposed sizing approach will be applied. The objective function of this problem consists of different operational costs such as energy production, operational and maintenance, startup and shutdown, emission, fuel costs, maintenance of spinning reserve and battery, which is one of the advantages of this paper. Furthermore, this paper studies a perfect set of operational constraints including, generating power limits, load demand balance, minimum up-time, minimum down-time, ramp rate capabilities, spinning reserve requirement and BESS operational constraints; that is the other advantage of the proposed method. The problem is formulated as a mixed integer linear programming (MILP) solved by the CPLEX solver in General Algebraic Model System (GAMS) software. Finally, to show the impact of the optimal size of BESS on the operational costs of MG, three different scenarios will be considered and compared with each other. The results show that the optimal size of the BESS exist and operational and total costs are minimum in the optimal case. As well as, output results compared by the other solvers such as MOSEK, LINDO confirmed the obtained results.

Keywords Optimal sizing, energy storage system, micro grid, unit commitment, operation cost.

Nomenclature

\[ P_{WT}^t \] Power generated by wind turbine at time \( t \) [kW].
\[ P_{WT}^{R} \] Rated power of wind turbine [kW].
\( v \) Wind speed [m/s].
\( v_{cut-in} \) Wind turbine cut-in speed [m/s].
\( v_{cut-out} \) Wind turbine cut-out speed [m/s].
\( v_r \) Wind turbine rated speed [m/s].
\[ P_{PV} \] Power generated by PV system [kW].
\( R \) Solar radiation [W/m²].
\( R_{STD} \) Standard solar radiation conditions that is usually set to 1000 W/m².
\( R_c \) Certain radiation point that usually is set to 150 W/m².


\( P_{PV} \) The rated power of PV system [kW].

\( f_{MT}^t \) Cost function of micro turbine at time t.

\( P_{MT}^t \) Power generated by micro turbine at time t [kW].

\( b_0^t, b_1^t \) Micro turbine cost function coefficients.

\( f_{FC}^t \) Cost function of fuel cell at time t.

\( P_{FC}^t \) Power generated by fuel cell at time t [kW].

\( c_0^t, c_1^t \) Fuel cell cost function coefficients.

\( W_{dis}^t \) Energy produced by diesel generator at time t [kWh].

\( P_{dis}^t \) Power generated by diesel generator at time t [kW].

\( \eta_{dis}^t \) Diesel generator efficiency.

\( f_{dis}^t \) Cost function of diesel generator at time t.

\( d_0^t, d_1^t \) The coefficients of the fuel consumption – power curve [L/kW].

\( TCBS \) Total cost of BESS per day [€ct/day].

\( \delta_{dis}^t \) Interest rate of BESS finance installations.

\( blt \) Lifetime of BESS.

\( FCBS \) Fixed cost of BESS [€ct].

\( O & MC_{BS} \) Operation and maintenance cost of BESS [€ct].

\( MGOC \) Operation cost of MG [€ct/day].

\( TC \) Total cost of MG [€ct/day].

\( SUC_{MT}^t \) Startup cost of diesel generator, FC, MT at time t [€ct].

\( SDC_{MT}^t \) Shutdown cost of diesel generator, FC, MT at time t [€ct].

\( \psi_{microgrid}^t \) Cost function of MG at time t.

\( SUC_{microgrid}^t \) Startup cost of MG at time t [€ct].

\( SDC_{microgrid}^t \) Shutdown cost of MG at time t [€ct].

\( O & MC_{microgrid}^t \) Operation and maintenance cost of MG at time t [€ct].

\( \rho_{dis}^t \) Consumed fuel cost rate of diesel generator, that is considered 6.5 €ct/L.

\( \tau_{dis}^t \) Spinning reserve at time t [kW].

\( \phi_{dis}^t \) Price of maintenance of spinning reserve, that is considered 1.2 €ct/kW.

\( \nu_{dis}^t \) A binary variable that indicates the on/off status of the diesel generator, MT and, FC at time t.

\( P_{MT}^t, P_{FC}^t \) Rated power of MT, FC, that is equal to maximum generated power of MT, FC.

\( P_{dis}^t, P_{MT}^t, P_{FC}^t, P_{WT}^t, P_{PV}^t \) Maximum limit of generated power of diesel generator, MT, FC, WT, PV.

\( P_{min}^t, P_{min}^t, P_{FC}^t, P_{MT}^t, P_{PV}^t \) Minimum limit of generated power of diesel generator, MT, FC, WT, PV.

\( \delta_{dis}^t \) Startup cost coefficients of diesel generator, MT and, FC.

\( \kappa_{dis}^t \) Shutdown cost coefficients of diesel generator, MT and, FC.

\( \sigma_{dis}^t \) Operation and maintenance cost of diesel generator, MT, FC, WT and, PV, respectively.

\( EC_{microgrid}^t \) Emission cost of MG at time t.

\( \gamma_j^t \) Price coefficient cost of pollutant j, including \( NO_x, SO_2, CO_2 \).

\( \beta_{ij}^t \) Emission factor of pollutant j by unit i including diesel generator, MT and, FC.

\( P_{BS}^t \) The BESS power at time t [kW].

\( P_{BS, ch}^t, P_{BS, dch}^t \) Charged and discharged power of BESS at time t [kW].

\( P_{max}^t, P_{max}^t \) Maximum rate of discharged and charged of BESS at time t [kW].

\( P_{min}^t, P_{min}^t \) Minimum rate of discharged and charged of BESS at time t [kW].

\( x_{\tau}, y_{\tau} \) Binary variables that represent the charge and discharge status.

\( W_{dis}^t \) Energy stored in BESS at time t [kWh].

\( \eta_{BS,ch}^t, \eta_{BS,dch}^t \) Initial and ending stored energy in BESS [kWh].

\( \eta_{BS,ch}^t, \eta_{BS,dch}^t \) Efficiency of charge and discharge.

\( P_D^t \) Load demand at time t [kW].

\( U_{dis}, U_{MT}, U_{FC}^t \) Minimum up-time of diesel generator, MT and, FC, respectively.

\( U_{0, dis}, U_{0, MT}, U_{0, FC} \) Minimum initial up-time of diesel generator, MT and, FC, respectively.

\( D_{dis}^t, D_{MT}^t, D_{FC}^t \) Minimum down-time of diesel generator, MT and, FC, respectively.

\( D_{0, dis}, D_{0, MT}, D_{0, FC} \) Minimum initial down-time of diesel generator, MT and, FC, respectively.

\( SU_{dis}, SU_{MT}, SU_{FC} \) Maximum startup ramps limit of diesel generator, MT and, FC, respectively.

\( SD_{dis}, SD_{MT}, SD_{FC} \) Maximum shutdown ramps limit of diesel generator, MT and, FC, respectively.

\( RU_{dis}, RU_{MT}, RU_{FC} \) Maximum ramps up limit of diesel generator, MT and, FC, respectively.

\( RD_{dis}, RD_{MT}, RD_{FC} \) Maximum ramps down limit of diesel generator, MT and, FC, respectively.

\( BSr \) BESS reserve at time t [kW].
\( \psi_{\text{dis},l} \) Slop of segment 1 of the piecewise linear approximation of diesel generator cost function.

\( P_{\text{dis},l} \) Generated power in segment 1 of the piecewise linear approximation of diesel generator cost function in period k.

\( H_{\text{dis},l} \) The upper limit of segment 1 of the piecewise linear approximation of diesel generator cost function.

1. Introduction

Nowadays, because of its benefits, micro grid (MG) is gradually becoming common. MG is formed by allocation of distributed resources, loads, energy storage systems and power electronic devices. According to some reasons (such as load demand, market price, location of DG deployment and etc.) MG can operate in grid-connected or off-grid mode [1-2]. Different systems can be used as energy storage system in MG. For example, some researchers used flywheel system [3], compressed air system [4], superconducting magnetic system [5], methane hyate gas system [6], hydrogen [7] and BESS [8] as energy storage system. Typically, two forms of cumulated and distributed BESS exist in MG [9]. Since the power production of photovoltaic system (PV), wind turbine (WT) and load forecasting are associated with uncertainty, presence of BESS is essential, especially in off-grid mode of operation [10].

Multiple studies have been done about considering the influence of the optimal size of BESS on cost reduction in MG. Some of these are presented here. In a study by Chen et al. [11], optimal sizing of the BESS problem in MG has been solved for both grid-connected and isolated modes. The used MG consists of WT, PV, MT, FC and BESS. Optimal sizing of BESS with thermal continuous power system has been studied by Chakraborty et al. [12]. Xiao et al. proposed a method for optimal sizing of BESS and economic dispatch of MG [13]. In that study, a two-stage strategy (i.e. mesh adaptive direct search (MADS) algorithm and improved particle swarm optimization (IPSO) algorithm) was respectively used to solve sizing and economic dispatch model. Sharma et al. recommended a cost based formulation for optimal sizing of BESS in the cost minimization of MG [14]. To solve the problem, the grey wolf algorithm was used. Minimizing the operational costs of MG by controlling local generations has been presented by Biazar et al. [15]. The objective function, including fuel cost of DGs, power exchanged by MG and customers, and startup and shutdown costs, was used in that study. A method based on genetic algorithm which was associated with an Energy Management Strategy (EMS) for optimal sizing of BESS to minimize the cost of MG was used in [16]. Optimal sizing of ESs for an isolated system with WT and wave energy resource, calculated in [17]. The real data for optimization problem has been obtained from three Canary islands. Besides of the role of optimal sizing of BESS in operational cost, BESS can help to manage the power and control of MG. To achieve this aim, some researchers have been done. A multi objective problem has been proposed for optimal operation of a MG in [18]. Objectives of problem were minimizing the operational cost and emission of MG, the problem was formulated non-linear. Koohi-kamali and et al. [19], proposed a power management system in a MG that protected the main grid from power changes of MG. That MG included diesel generator, PV and BESS. BESS was used for supporting frequency control in MG which was suggested in [20]. The aims of this control process were increasing the stability and reliability of power supply. In this paper, a cost-based method for optimal sizing of BESS in MG with respect to operational cost reduction is proposed. Generally, since there is no direct mathematical relationship between the size of the battery and other resources of MG, the unit commitment problem should be solved. Solving this problem gives us both generated power and operational cost of MG units. Now, we can determine the optimal size of BESS in MG. Unit commitment problem is a non-convex and nonlinear one. The reason of its non-convexity is the binary nature of the on/off variables and status of units; moreover, nonlinearity is caused by power production curves. Thus, solving the unit commitment problem is hard [21]. Evolutionary algorithms such as GA, PSO and so on are dependent on the initial population and number of simulating iterations. Thus the simulation time is increased. On the other hand, along with increasing in the size of problem, its coding becomes difficult, and the previously said algorithms may be trapped in local optimum solutions. Recently, due to progress in mixed integer linear programing (MILP) solvers, this formulation is used because of its accuracy in solution and ease of programming [22].

This paper studies a perfect set of costs and constraints. The objective function of the problem encompasses different costs that are constrained by different limitations. To solve the unit commitment problem, a mathematical MILP formulation solved by CPLEX in GAMS software is proposed. The simulation speed of GAMS is high so it is one of the best software for solving the complex and large problems [23].

The rest of the paper is organized as follows. Section 2 introduces the structure studied MG and describes its elements. Section 3 introduces the proposed method, that is divided into objective function with constraints and solving method subsections. The simulation and results are...
represented in Section 4. Finally, the conclusion is given in Section 5.

2. Description of MG’s Structure and its Components.

The studied MG is a low voltage system that works independently of the utility grid (off-grid). Therefore, the frequency, voltage and load demand should be controlled. MG has a WT, PV, micro turbine (MT), fuel cell (FC), diesel generator, BESS and a number of loads. These loads divided into residential, industrial and commercial ones. Figure 1 shows the studied MG. The MG elements have been explained in following subsections.

2.1. Wind Turbine (WT)

Wind energy is one of the renewable energy sources [24]. It is obtained from pressure differences across the earth, which is due to Heterogeneous radiation of the sun [25]. WT is a electromechanical device that transforms the kinetic energy to electric power [26, 27]. Generally, according to rotation axis directions, WT is divided into two categories, i.e. vertical axis and the horizontal axis [28]. WT produces power without pollution [29]. The output power of WT, that is not constant and related to the wind speed fluctuations, is calculated as follows [30,31]:

\[
P_{WT}^f = \left\{ \begin{array}{ll}
0 & \text{if } \frac{v}{v_{cut-in}} > R_f \\
R_f^v \left( \frac{v^3}{v_{cut-in}^3} - \frac{v^3}{v_{cut-in}^3} \right) & \text{if } \frac{v_{cut-in}}{v_{cut-in}} < \frac{v}{v_{cut-out}} < 1 \\
R_f^v & \text{if } \frac{v_{cut-out}}{v_{cut-out}} < \frac{v}{v_{cut-out}} < 1 \\
0 & \text{if } \frac{v}{v_{cut-out}} > 1
\end{array} \right.
\] (1)

2.2 Photovoltaic System (PV)

Today, because of their advantages, using PV systems is common. These advantages are soundless, clean-emission, no fuel consumption, the possibility of utilizing them in most places [32] and easy installation [33]. But, currently, the price of electricity supplied from a PV system is more than the utility electricity price [34]. PV modules consist of a number of cells that convert sunlight energy to electric power. Modules can be connected in series and parallel modes. PV panels are made from a combination of PV modules; and the PV array are defined as the linkage of a number of PV panels [35]. The output power of the PV system is related to the radiation affected by environmental conditions [36, 33]. Therefore, the output power of a PV system can be calculated as follows [30]:

\[
P_{PV}^R = \left\{ \begin{array}{ll}
P_{PV}^0 \left( \frac{R^2}{R_{STD}^2 - R} \right) & \text{if } 0 \leq R \leq R_c \\
P_{PV}^0 \left( \frac{R}{R_{STD}} \right) & \text{if } R_c \leq R \leq R_{STD} \\
0 & \text{if } R_{STD} \leq R
\end{array} \right.
\] (2)

2.3. Micro Turbine (MT)

MT is one of the distributed generation resources. MT can work in two modes, grid-connected and off-grid modes. MTs, compared to other DGs, have premiums, including high revenue, low inertia and fast response to the standard gas turbines. MTs can consume a variety of fuels such as natural gas, diesel, hydrogen, propane, etc [37]. The rated power of MTs can vary from 25 kW to 500 kW [38]. In any time, the cost function of MT consists of fuel cost and fixed cost (installation) which can be written as [11]:

\[
f_{MT}^f = b_0 P_{MT}^f + b_1
\] (3)

2.4. Fuel cell (FC)

FC is an electrochemical system that transforms chemical energy of fuel directly into electrical energy [39]. According to used electrolyte type and their temperature, FCs are classified into two groups: low temperature like PEMFCs (proton exchange membrane fuel cells) and SOFCs (solid-oxide fuel cells) [40, 41]. As stated in MT section, the fuel cost and fixed cost for FCs can be formulated as [11]:

\[
f_{FC}^f = c_0 P_{FC}^f + c_1
\] (4)

2.5. Diesel Generator

A diesel generator is formed by joining a diesel engine and synchronous generator on a same axis. Diesel generators, beside power supply, can operate as back-up and emergency source for key installations, including hospitals, airports and etc [42]. In the off-grid MGs, in addition to generating power, diesel generators can contribute to frequency regulation. In any time, the energy generated by diesel generators with nominal power can be stated as [43]:

\[
W_{dis}^f = P_{dis}^f \eta_{dis}
\] (5)

On the other hand, fuel cost and installation cost of diesel generator and fuel consumption are attained as (6), (7):
battery as Eq. (15). (1 )   

\[
TCBS = \frac{W_{BS}}{365} \left( \frac{ir(1+ir)^{ht}}{(1+ir)^{ht}-1} \right) (FC_{BS} + O & MC_{BS})
\] (15)

3. Proposed Method

3.1. Objective Function

As mentioned in the previous sections, the objective function in this paper consists of fuel cost and installation cost as Eqs. (3), (4), (6), startup and shutdown costs of units as Eqs. (21), (22), operation and maintenance costs as Eq. (23), air pollution cost as Eq. (24), fuel consumption of diesel generator as Eq. (5), reserve cost mentioned in Eq. (17), fixed and variant costs of the battery as Eq. (15). Thus the mathematical formulation of objective function can be written as follows:

\[
TC = MGOC + TCBS
\] (16)

\[
\text{Minimize } TC = \sum_{i=1}^{n} \left( f_i^{t} + f_i^{MT} + f_i^{FC} \right) + \delta_{i}^{t}\psi_{i}\gamma_{i} + \delta_{i}^{t}\psi_{i}\gamma_{i} + \delta_{i}^{t}\psi_{i}\gamma_{i}
\] (17)

\[
SUC_{microgrid} = SUC_{dis}^{t} + SUC_{MT}^{t} + SUC_{FC}^{t}
\] (19)

\[
SUC_{FC} = SUC_{microgrid}^{t} + SUC_{MT}^{t} + SUC_{FC}^{t}
\] (20)

\[
EC_{microgrid}^{t} = \sum_{j=1}^{m} \left( \sum_{k=1}^{n} \beta_{jk} \right) = P_{d0}^{t} (\gamma_{d0} \psi_{d0} + \gamma_{d0} \psi_{d0}) + P_{d1}^{t} (\gamma_{d1} \psi_{d1} + \gamma_{d1} \psi_{d1})
\] (24)

The proposed problem is subjected to the following constraints:

3.1.2. Power Generation Limit

Since each unit doesn’t produce any power value, it must be limited by upper and lower bounds. In this paper, diesel generator, MT, FC, WT, PV produce power between minimal and maximal limits. This is formulated as:

\[
P_{\text{min}}^{dis} \leq P_{\text{dis}}^{l} \leq P_{\text{max}}^{dis}
\] (25)

\[
P_{\text{min}}^{MT} \leq P_{\text{MT}}^{l} \leq P_{\text{max}}^{MT}
\] (27)

\[
P_{\text{min}}^{PV} \leq P_{\text{PV}}^{l} \leq P_{\text{max}}^{PV}
\] (29)

3.1.3. BESS Constraints

When the sum of generated power of units is more than load demands, battery is charged. However, if the generated power of units is less than load demands, battery is discharged to MG to supply shortages. The BESS constraints consist of maximal and minimal charge and
discharge limits as Eqs. (31) and (32), lack of simultaneously charging and discharging operation Eq. (33), available energy at every time in BESS Eq. (34), energy capacity limits Eq. (35) and equality of starting and ending energy Eq. (36). This constraints are expressed as follows:

\[ P_{BS}^{t} = P_{BS,ch}^{t} + P_{BS,dch}^{t} \]  
\[ 0 \leq P_{BS,ch}^{t} \leq P_{BS,ch}^{max} \]  
\[ 0 \leq P_{BS,dch}^{t} \leq P_{BS,dch}^{max} \]  
\[ x^{t} + y^{t} \leq 1 \]  
\[ W_{BS}^{t} = W_{BS,ch}^{t} - (U_{BS,ch}^{t} \Delta t) \]  
\[ W_{BS}^{min} \leq W_{BS}^{t} \leq W_{BS}^{max} \]  
\[ W_{BS}^{0} = W_{BS}^{t} \]  
\[ 3.1.4. \ Load \ Demand \ Balance \]

The sum of the generated powers by units and BESS can be supply the load demand:

\[ P_{dis}^{t} = P_{MT}^{t} + P_{PV}^{t} + P_{BS}^{t} = P_{D}^{t} \]  
\[ 3.1.5. \ Minimum \ Up-Time \]

The minimum up-time (MUT) denotes that if a unit is on it must be on for a specified minimum time. In this case, this restriction is regarded for diesel generator, MT, and FC. This restriction can be formulated as follows [50]:

\[ t_{dis}^{i} \in [0,1] \]  
\[ \sum_{k=0}^{T} I_{dis}^{k} \geq U_{dis}^{0} - l_{dis}^{k-1} - I_{dis}^{k-1} \]  
\[ t = U_{dis}^{0} + 1, \ldots, T - U_{dis} + 1 \]  
\[ I_{dis}^{0} = \sum_{k=0}^{T} I_{dis}^{k} \]  
\[ \sum_{k=0}^{T} I_{MT}^{k} \geq U_{MT}^{0} - l_{MT}^{k-1} - I_{MT}^{k-1} \]  
\[ t = U_{MT}^{0} + 1, \ldots, T - U_{MT} + 1 \]  
\[ I_{MT}^{0} = \sum_{k=0}^{T} I_{MT}^{k} \]  
\[ \sum_{k=0}^{T} I_{FC}^{k} \geq U_{FC}^{0} - l_{FC}^{k-1} - I_{FC}^{k-1} \]  
\[ t = U_{FC}^{0} + 1, \ldots, T - U_{FC} + 1 \]  
\[ I_{FC}^{0} = \sum_{k=0}^{T} I_{FC}^{k} \]  
\[ 3.1.6. \ Minimum \ Down-Time \]

As stated in the previous section, the minimum down-time (MDT) represents that if a unit is off it must be off for a specified minimum time. In this paper, this restriction is regarded for diesel generator, MT and FC. This restriction can be formulated as follows [50]:

\[ t_{dis}^{i} \in [0,1] \]  
\[ \sum_{k=0}^{T} I_{dis}^{k} \geq D_{dis}^{0} - l_{dis}^{k-1} - I_{dis}^{k-1} \]  
\[ t = D_{dis}^{0} + 1, \ldots, T - D_{dis} + 1 \]  
\[ D_{dis}^{0} = \sum_{k=0}^{T} I_{dis}^{k} \]  
\[ \sum_{k=0}^{T} I_{MT}^{k} \geq D_{MT}^{0} - l_{MT}^{k-1} - I_{MT}^{k-1} \]  
\[ t = D_{MT}^{0} + 1, \ldots, T - D_{MT} + 1 \]  
\[ D_{MT}^{0} = \sum_{k=0}^{T} I_{MT}^{k} \]  
\[ \sum_{k=0}^{T} I_{FC}^{k} \geq D_{FC}^{0} - l_{FC}^{k-1} - I_{FC}^{k-1} \]  
\[ t = D_{FC}^{0} + 1, \ldots, T - D_{FC} + 1 \]  
\[ D_{FC}^{0} = \sum_{k=0}^{T} I_{FC}^{k} \]  
\[ 3.1.7. \ Ramp \ Capabilities \]

Ramp constraints say that, although, when there are needs for power, units cannot aggravate their production to any amount, they can do so according to their ramp rate. The first constraint represents that if the unit (i) is on then its maximum available power at time (t) \((P_{i}^{t,max})\) can not be greater than sum of the produced power exactly at time (t-1) and the amount of its ramp up [50].

\[ P_{dis}^{max} \leq P_{dis}^{t-1} + R U_{dis}^{t-1} \lambda_{dis}^{t} + S U_{dis}^{t-1} (I_{dis}^{t-1} - I_{dis}^{t}) + P_{dis}^{t} (1-I_{dis}^{t}) \]  
\[ P_{MT}^{max} \leq P_{MT}^{t-1} + R U_{MT}^{t-1} \lambda_{MT}^{t} + S U_{MT}^{t-1} (I_{MT}^{t-1} - I_{MT}^{t}) + P_{MT}^{t} (1-I_{MT}^{t}) \]  
\[ P_{FC}^{max} \leq P_{FC}^{t-1} + R U_{FC}^{t-1} \lambda_{FC}^{t} + S U_{FC}^{t-1} (I_{FC}^{t-1} - I_{FC}^{t}) + P_{FC}^{t} (1-I_{FC}^{t}) \]

The second constraint indicates the down ramping of units. In this case, if the unit is on, generated power of units at time (t) is more than generated power at time (t-1) minus the ramp down. while the unit is off, generated power at time (t) is more than generated power at time (t-1) minus the maximum shutdown limit.

\[ P_{dis}^{t} \geq P_{dis}^{t-1} - R D_{dis}^{t} \lambda_{dis}^{t} - S D_{dis}^{t} (I_{dis}^{t-1} - I_{dis}^{t}) - P_{dis}^{t} (1-I_{dis}^{t}) \]  
\[ P_{MT}^{t} \geq P_{MT}^{t-1} - R D_{MT}^{t} \lambda_{MT}^{t} - S D_{MT}^{t} (I_{MT}^{t-1} - I_{MT}^{t}) - P_{MT}^{t} (1-I_{MT}^{t}) \]  
\[ P_{FC}^{t} \geq P_{FC}^{t-1} - R D_{FC}^{t} \lambda_{FC}^{t} - S D_{FC}^{t} (I_{FC}^{t-1} - I_{FC}^{t}) - P_{FC}^{t} (1-I_{FC}^{t}) \]

And the final constraint expresses that, in the next period, when the unit goes off, the maximum shutdown ramp limit is more than the maximum available power at the present time.

\[ P_{dis}^{t+1,max} \leq S D_{dis}^{t} (V_{dis}^{t+1} - V_{dis}^{t}) + P_{dis}^{t} (I_{dis}^{t+1} - I_{dis}^{t}) \]  
\[ P_{MT}^{t+1,max} \leq S D_{MT}^{t} (V_{MT}^{t+1} - V_{MT}^{t}) + P_{MT}^{t} (I_{MT}^{t+1} - I_{MT}^{t}) \]  
\[ P_{FC}^{t+1,max} \leq S D_{FC}^{t} (V_{FC}^{t+1} - V_{FC}^{t}) + P_{FC}^{t} (I_{FC}^{t+1} - I_{FC}^{t}) \]

The maximum available power of each unit can be written as follows:

\[ P_{i}^{max} \leq P_{i}^{t} \leq P_{i}^{t,max} \]
\begin{equation}
0 \leq P_{i,r}^{t},_{\text{max}} \leq P_{i,r}^{t},_{i}
\end{equation}

### 3.1.8. Spinning Reserve

To improve the reliability and security of MG, spinning reserve is considered [51]. This constraint can be written as follows [16]:

\begin{equation}
(P_{\text{dis}}^{t} - P_{\text{dis}}^{t,r}) + (P_{\text{MT}}^{t} - P_{\text{MT}}^{t,r}) + (P_{\text{FC}}^{t} - P_{\text{FC}}^{t,r}) + B_{S}^{t,r} \geq r^{t}
\end{equation}

\begin{equation}
B_{S}^{t,r} = \begin{cases}
P_{\text{max}} - P_{B}^{t} & \text{if } W_{i}^{t-1} \geq W_{i}^{t}_{,\text{min}} \\
0 & \text{if } W_{i}^{t-1} < W_{i}^{t}_{,\text{min}}
\end{cases}
\end{equation}

### 3.2. Solving Method

As stated in previous sections, for optimal sizing of BESS, with regard to operation cost minimization in MG, a number of processes must be done. In the first step, the initial values of the problem should be determined. Objective functions and constraints of the problem are formulated as mixed integer linear programming (MILP). Unit commitment problem and economic dispatch are solved with the CPLEX in GAMS software. After using the CPLEX, the optimum schedule, optimum power generated by units and MGOC cost are obtained. It must be mentioned that the time horizon of the optimal sizing problem is one day. According to Eq. (35), in the next stage, \(\Delta W_{BS}^{\text{max}}\) is added to \(W_{BS}^{\text{max}}\) that is stated as \((W_{BS}^{\text{max}} + \Delta W_{BS}^{\text{max}})\). Then the previous steps must be repeated and MGOC and TCBS must be also computed. This procedure is iterated until the operational cost becomes fixed and TC be minimized. Finally, the optimal sizing of BESS is the point where the TC curve is minimized. After the optimum point, the cost begins to increase. Figure 3 shows the flowchart of solving process of the problem. Figure 4 indicates the MGOC, TCBS, and TC curve with optimum point.

### 4. Simulation and Results

The proposed method is applied to MG (Fig. 1). Load demand curve, WT and PV forecasted power curves are shown in Figures 5, 6, and 7. The data of these curves are obtained from [10]. Cost coefficients and power limits of diesel generator, MT, FC, WT, and PV are introduced in Tables 1, 2, 3, and 4, respectively. The operational and maintenance costs of units in the Tables are obtained from [49]. Emission factors of units are given in Table 5. Diesel generator emission factor is extracted from [52] and the same factor of MT, FC are derived from [30]. Ramp coefficients, MUT and MDTs are listed in Table 6. The spinning reserve magnitude is set to 10% of load demand in each time period. Fixed and installation, operation and maintenance costs of BESS are 465 (Ect/kWh) and 15 (Ect/kWh), respectively. Also, lifetime and interest rate of BESS are considered as 3 and 0.06 [10,49]. The maximum rate of charge and discharge of BESS is 20 kW. The initial and final stored charge of BESS are the same and considered 10 kWh. Charging and discharging efficiency is assumed 0.95. Minimum of WB \((WB_{BS}^{\text{min}})\) is 5 kWh and its maximum \((WB_{BS}^{\text{max}})\) is determined by the proposed method. Maximum available power of diesel generator, MT, and FC in each time period is presumed 1.05 times as much as power generated by units in each time period. To discuss about the effect of BESS on operation and total costs of MG, three scenarios are considered. We tried these scenarios involved optimum sizes, and the size
which is less than optimum one, and the state in which there is no battery.

4.1. Scenario 1

In this scenario, MG operates without BESS. The generated power of units in this scenario must supply the load demands without BESS. every diesel generator, MT, FC produces power according to its costs with regard to its constraints. Based on simulation results, total cost of operation is calculated as 1523.48 (€ct) and the optimal power generated is shown in Fig. 8. As can be seen in this figure, diesel generator is off for 5 hours, and begins to produce power in the 6th hour; that implies the MUT and MDT are satisfied. Other units are on and produce power all times. As seen in Fig. 8, the diesel generator plays as supportive in studied MG.

4.2. Scenario 2.

In this case, the size of BESS is regarded 50 kWh. BESS contributes in supplying load demand. When the energy in MG is excess, the BESS stores it and when required, releases the energy. With respect to the constraints and costs, it may not be the case that
BESS is fully charged and discharged. BESS may have less contribution in supplying power. This contribution in the MG causes reduction in costs. In this scenario, the operational cost of the MG is 1367.49 (€ct) and the total cost is 1392.09 (€ct). As can be seen, adding BESS to MG reduces the costs. In this case, compared to Scenario 1, the operational and total costs are reduced 155.99 and 131.09 (€ct), respectively, which is almost equal to 10.23% and 8.62% in one day. In this scenario, Figures 9 and 10 demonstrate optimal generated power of units and energy stored in BESS, respectively. According to Fig. 9, when the load demand is on peak value and electrical power is needed, the BESS begins to be discharged and energy stored in BESS decreases. Also when the load is low, BESS is charged and the stored energy increases. On the other hand, due to the costs, the diesel generator is off for 15 hours and it begins to generate power at 16th hour.

### Table 1. Diesel generator cost coefficients and power limits.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(a_0)</th>
<th>(a_1)</th>
<th>(a_2)</th>
<th>(O &amp; MC) (€ct / kWh)</th>
<th>(\delta) (€ct)</th>
<th>(\kappa) (€ct)</th>
<th>(P_{\text{min}}) (kW)</th>
<th>(P_{\text{max}}) (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diesel</td>
<td>0.074</td>
<td>0.2333</td>
<td>0.4333</td>
<td>0.1525</td>
<td>1.7</td>
<td>1.7</td>
<td>2</td>
<td>40</td>
</tr>
</tbody>
</table>

### Table 2. MT cost coefficients and power limits.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(b_0)</th>
<th>(b_1)</th>
<th>(O &amp; MC) (€ct / kWh)</th>
<th>(\delta) (€ct)</th>
<th>(\kappa) (€ct)</th>
<th>(P_{\text{min}}) (kW)</th>
<th>(P_{\text{max}}) (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MT</td>
<td>0.321</td>
<td>0.013</td>
<td>0.0446</td>
<td>0.96</td>
<td>0.96</td>
<td>6</td>
<td>35</td>
</tr>
</tbody>
</table>

### Table 3. FC cost coefficients and power limits.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(c_0)</th>
<th>(c_1)</th>
<th>(O &amp; MC) (€ct / kWh)</th>
<th>(\delta) (€ct)</th>
<th>(\kappa) (€ct)</th>
<th>(P_{\text{min}}) (kW)</th>
<th>(P_{\text{max}}) (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FC</td>
<td>0.215</td>
<td>0.015</td>
<td>0.0862</td>
<td>1.65</td>
<td>1.65</td>
<td>3</td>
<td>25</td>
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</tbody>
</table>

### Table 4. WT and PV cost coefficients and power limits.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(c_0)</th>
<th>(c_1)</th>
<th>(O &amp; MC) (€ct / kWh)</th>
<th>(\delta) (€ct)</th>
<th>(\kappa) (€ct)</th>
<th>(P_{\text{min}}) (kW)</th>
<th>(P_{\text{max}}) (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WT</td>
<td>0</td>
<td>0</td>
<td>0.525</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>PV</td>
<td>0</td>
<td>0</td>
<td>0.2082</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>25</td>
</tr>
</tbody>
</table>

### Table 5. The pollution components and emission factors of pollution.

<table>
<thead>
<tr>
<th>Pollution components</th>
<th>(\gamma) (€ct / kg)</th>
<th>(\beta_{\text{dis}}) (kg / kWh)</th>
<th>(\beta_{\text{MT}}) (kg / kWh)</th>
<th>(\beta_{\text{FC}}) (kg / kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(NO_x)</td>
<td>10.0714</td>
<td>0.0218</td>
<td>0.00003</td>
<td>0.00044</td>
</tr>
<tr>
<td>(SO_2)</td>
<td>2.3747</td>
<td>0.000454</td>
<td>0.000006</td>
<td>0.0000088</td>
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<tr>
<td>(CO_2)</td>
<td>0.0336</td>
<td>0.001432</td>
<td>0.001078</td>
<td>0.001598</td>
</tr>
</tbody>
</table>

### Table 6. Ramp parameters and MUT/MDTs of units.

<table>
<thead>
<tr>
<th>Unit</th>
<th>(RU)</th>
<th>(SU)</th>
<th>(RD)</th>
<th>(SD)</th>
<th>(U_i)</th>
<th>(D_i)</th>
<th>(U_0)</th>
<th>(D_0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diesel</td>
<td>10</td>
<td>4</td>
<td>10</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

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Table 7. Comparison of scenarios.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Operational cost (€ct)</th>
<th>Total cost (€ct)</th>
<th>Operational cost reduction (%)</th>
<th>Total cost reduction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1</td>
<td>1523.48</td>
<td>1523.48</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>1367.49</td>
<td>1392.09</td>
<td>10.23</td>
<td>8.62</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>1305.76</td>
<td>1357.42</td>
<td>14.29</td>
<td>10.9</td>
</tr>
</tbody>
</table>

Fig. 9. Generated power by units in Scenario 2.

Fig. 10. Stored energy in BESS in Scenario 2.

Fig. 11. Generated power by units in Scenario 3.

Fig. 12. Stored energy in BESS in Scenario 3.

4.3. **Scenario 3**

In this scenario, the optimal point is obtained by trade-off between MGOC, TCBS curves and different sizes of BESS. In accordance with Fig. 4, the optimal size of BESS with regard to cost is 105 kWh. In this condition, operational and total cost of MG will be 1305.76 (€ct) and 1357.42 (€ct), respectively. As can be seen, compared to the costs obtained in Scenario 1, the calculated operational and total costs are reduced 217.72 (€ct) and 166.06 (€ct), that is equal to 14.29% and 10.9% reduction in cost. Compared this scenario to Scenario 2, the operational and total costs of the MG are reduced 61.73 (€ct) and 34.67 (€ct).
5. Conclusion

In this paper, because of disconnecting MG from main grid, presence of BESS in power management is urgent. The purpose of this paper is to determine the optimal size of BESS in MG by considering the operation costs of MG. For this aim, an analytical cost based method is proposed. Our proposed method applied to a off-grid MG including diesel generator, MT, FC, WT, PV and BESS. In the objective function, we simultaneously considered different operational costs considering energy production, operational and maintenance, startup and shutdown, emission, fuel costs, maintenance spinning reserve of units and battery. At the same time, we had to keep in view various constraints for the units and BESS; that is one of the advantages of this paper. Although, the feasibility of problem was difficult, the problem became feasible by using actual data of units and BESS parameters. The problem was formulated as MILP problem and solved by CPLEX system. Because of complexity of model and binary variables in formulation of the problem, MILP formulation is more effective than the other algorithms. On the other hand, software gives exact solution for unit commitment problem, and simulation time is increasingly decreased; that is another advantage of our work than the others. Furthermore, the problem was solved with MOSEK and LINDO systems that verified the results. Presence of BESS helped to minimize the cost and enhance the reliability of MG. The optimal size was obtained according to Fig. 6. Finally, a number of scenarios have been considered and discussed. Results show that, when the size of BESS is increased, operational cost is reduced but BESS costs go up. According to considered scenarios, in the first one, we had no cost minimization and the total cost was 1523.48 (€ct). In the second, operational and total costs were reduced 10.23% and 8.62%, respectively. In the optimum size of BESS, the operational and total costs were reduced more than the other scenarios, which were about 14.29% and 10.9%, respectively. By increasing the size of BESS more than the optimum size, the operational costs of MG, in accordance with MG operation itself, almost moved to be constant but, at the same time, BESS costs increase. As a result, the final costs increase.

Reference


